

Georgia Performance Standard(s) MM3D1

Your Notes

6.1 Construct and Interpret Binomial Distributions

Goal • Study probability distributions.

Example 1 Construct a probability distribution

Let X be a random variable that represents the sum when two four-sided dice are rolled. Make a table and a histogram showing the probability distribution for X.

Solution

The possible values of X are the integers from 2 to 8. The table shows the number of outcomes for each value of X. Divide the number of outcomes for X by the total number of outcomes ______ to get P(X).



X (sum)	2	3	4	5	6	7	8
Outcomes	1	2	3	4	3	2	1
P(X)							

Example 2 Inte

Interpret a probability distribution

Use the probability distribution in Example 1 to answer each question. (a) What is the most likely outcome of rolling the two dice? (b) What is the probability that the sum of the two dice is at most 4?

Solution

- **a.** The most likely outcome of rolling the two dice is the value of *X* for which P(X) is greatest. This probability is greatest for X =____. So, the most likely outcome of rolling the two dice is a sum of ____.
- **b.** The probability that the sum of the two dice is at most 4 is:



Checkpoint Complete the following exercise.

1. Let *X* be the letter on a letter block randomly chosen from a bag containing 7 blocks labeled "A," 3 blocks labeled "B," 6 blocks labeled "C," and 5 blocks labeled "D." Make a table and a histogram showing the probability distribution for *X*.

X		
Outcomes		
P(X)		



BINOMIAL EXPERIMENTS

A binomial experiment meets the following conditions:

- There are *n* trials.
- Each trial has only two possible outcomes: ______
 and ____.
- The probability for success is the _____ for each trial. This probability is denoted by p. The probability for failure is given by 1 p.

For a binomial experiment, the probability of exactly k successes in n trials is:

P(k successes) =

found that 68% of the students are not afraid to Suppose you randomly su 5 students. Draw a histog of the binomial distribution for your survey.	chool o fly. Irvey gram on
The probability that a rand selected student is not afr to fly is $p = _$. Becau	aid Number of students not afraid to fly use
P(k = 0) =	¯' ≈
P(k = 1) =	≈
P(k = 2) =	≈
P(k = 3) =	≈
P(k=4) =	≈
P(k = 5) =	≈
Use the binomial distribut	tion in Example 3. outcome of the survey?
b. What is the probability	that at least 5 students are no
 b. What is the probability afraid to fly? 	
 b. What is the probability afraid to fly? c. Describe the shape of the shape o	the binomial distribution.
 b. What is the probability afraid to fly? c. Describe the shape of the solution 	the binomial distribution.
 b. What is the probability afraid to fly? c. Describe the shape of the shape o	the binomial distribution. The of the survey is the value of atest. This probability is greates ost likely outcome is that o afraid to fly.
 b. What is the probability afraid to fly? c. Describe the shape of the shape o	the binomial distribution. The of the survey is the value of atest. This probability is greates ost likely outcome is that of afraid to fly.
b. What is the probability afraid to fly? c. Describe the shape of t Solution a. The most likely outcom <i>k</i> for which $P(k)$ is great for $k = _$. So, the mo- the 5 students are not b. The probability that $k \ge$ $P(k \ge 3) = _$	the binomial distribution. The of the survey is the value of atest. This probability is greates ost likely outcome is that of afraid to fly. 2 3 is:

Your Notes

Checkpoint Use the following information to complete the exercises. In a survey of your neighborhood, 57% of the families own a pet. Suppose you randomly survey 6 families.



6.1 Practice

Decide whether the random variable X is discrete or continuous. *Explain*.

- 1. X represents the amount of time it takes to download a file from the Internet.
- 2. X represents the number of fish caught during a fishing tournament.

Make a table and a histogram showing the probability distribution for the random variable. *Describe* the distribution as either *symmetric* or *skewed*.

3. B = the number on a table tennis ball randomly chosen from a bag that contains 4 balls labeled "1," 4 balls labeled "2," and 2 balls labeled "3."



4. D = the absolute value of the difference when two six-sided dice are rolled.



In Exercises 5–8, use the given histogram of a probability distribution for a random variable *X*.

- **5.** What is the probability that *X* is equal to 1?
- **6.** What is the least likely value for *X*?
- **7.** What is the probability that *X* is at most 3?
- **8.** *Describe* the shape of the distribution.



6.1 **Practice** continued

Calculate the probability of tossing a coin 15 times and getting the given number of heads.

9. 4 **10.** 7 **11.** 10 **12.** 2

Calculate the probability of randomly guessing the given number of correct answers on a 20-question multiple choice exam that has choices A, B, C, and D for each question.

13.	5	14. 10	15. 15	16.	20

- **17.** Automobile Accidents An automobile-safety researcher claims that 1 in 10 automobile accidents are caused by driver fatigue. What is the probability that at least three out of five automobile accidents are caused by driver fatigue?
- **18.** Pet Allergies An analyst claims that about 70% of U.S. households own either a cat or a dog, and an estimated 10% of the U.S. population is allergic to animals.
 - **a.** What is the probability that exactly 1 person in a class of 20 students owns either a cat or a dog? (Assume that no two students in the class come from the same household.)
 - **b.** What is the probability that at most 3 students in a class of 20 students are allergic to animals?

6.2 Use Normal Distributions



MM3D2a,

MM3D2b

Your Notes

Goal • Study normal distributions.

VOCABULARY

Normal distribution

Normal curve

Standard normal distribution

z-score

AREAS UNDER A NORMAL CURVE

A normal distribution with mean \overline{x} and standard deviation σ has these properties:

- The total area under the related normal curve is
- About <u>%</u> of the area lies within 1 standard deviation of the mean.
- About ____% of the area lies within 2 standard deviations of the mean.
- About ____% of the area lies within 3 standard deviations of the mean.



Your Notes

Example 1 Find a normal probability A normal distribution has mean \bar{x} and standard deviation σ . For a randomly selected *x*-value from the distribution, find $P(\overline{x} \le \overline{x} \le \overline{x} + 2\sigma)$ $\overline{X} + \sigma$ σ $|\mathbf{x}|$ Зд 2_{σ} 2_{σ} 39 Solution | | |x |x $|\times|$ The probability that a randomly selected *x*-value lies between ____ and ____ is the shaded area under the normal curve. Therefore: $P(\bar{x} \le x \le \bar{x} + 2\sigma) = + =$

Checkpoint Complete the following exercise.

1. A normal distribution has mean \overline{x} and standard deviation σ . For a randomly selected *x*-value from the distribution, find $P(x \le \overline{x} - \sigma)$.

Example 2 Interpret normally distributed data

Math Scores The math scores of an exam are normally distributed with a mean of 518 and a standard deviation of 115.



- a. About what percent of the Math Score: test-takers have scores between 518 and 748?
- **b.** About what percent of the test-takers have scores less than 403?

Solution

- a. The scores of 518 and 748 represent _____ standard deviations to the _____ of the mean. So, the percent of the test-takers that have scores between 518 and 748 is ___% + ___% = ___%.
- **b.** A score of 403 is ______ standard deviation to the ______ of the mean. So, the percent of the test-takers that have scores less than 403 is % + % + % = %.

Example 3 Use a z-score and the standard normal table

Height A survey of a group of women found that the height of the women is normally distributed with a mean height of 64.5 inches and a standard deviation of 2.5 inches. Find the probability that a randomly selected woman is at most 58 inches tall.

Solution

1. Find the *z*-score corresponding to an *x*-value of 58.

 $z = \frac{x - \bar{x}}{\sigma} = \qquad \qquad = _$

2. Use the standard normal table to find $P(x \le 58) = P(z \le$ _____). The table shows that $P(z \leq ___) = ___$. So, the probability that a randomly selected woman is at most 58 inches tall is _____.

z	.0	.1	.2	.3	.4
-2	.0228	.0179	.0139	.0107	.0082
2	.9772	.9821	.9861	.9893	.9918
z	.5	.6	.7	.8	.9
-2	.0062	.0047	.0035	.0026	.0019
2	.9938	.9953	.9965	.9974	.9981

Checkpoint Complete the following exercises.

2. In Example 2, about what percent of the test-takers have scores between 403 and 633?

Homework

3. In Example 3, find the probability that a randomly selected woman is at most 70 inches tall.

6.2 Practice

A normal distribution has mean \overline{x} and standard deviation σ . Find the indicated probability for a randomly selected *x*-value from the distribution.

1. $P(x \ge \bar{x})$ **2.** $P(x \le \bar{x} - 2\sigma)$ **3.** $P(x \le \bar{x} + 3\sigma)$

Give the percent of the area under the normal curve represented by the shaded region.



A normal distribution has a mean of 18 and a standard deviation of 3. Find the probability that a randomly selected *x*-value from the distribution is in the given interval.

6.	Between 18 and 21	7.	Between 12 and 18	8.	Between 15 and 24
----	-------------------	----	-------------------	----	-------------------

9. At least 21

10. At least 27

11. At most 12

6.2 **Practice** continued

A normal distribution has a mean of 50 and a standard deviation of 5. Use the standard normal table to find the indicated probability for a randomly selected *x*-value from the distribution.

- **12.** $P(x \le 50)$ **13.** $P(x \le 55)$ **14.** $P(x \le 40)$
- **15.** $P(x \le 62)$ **16.** $P(x \le 47)$ **17.** $P(x \le 34)$

In Exercises 18 and 19, use the following information.

Restaurant Seating A restaurant is busiest Saturday from 5:00 P.M. to 8:00 P.M. During these hours, the waiting time for customers in groups of 4 or less to be seated is normally distributed with a mean of 15 minutes and a standard deviation of 2 minutes.

- **18.** What is the probability that customers in groups of 4 or less will wait 9 minutes or less to be seated during the busy Saturday night hours?
- **19.** What is the probability that customers in groups of 4 or less will wait 17 minutes or more to be seated during the busy Saturday night hours?

In Exercises 20 and 21, use the following information.

Light Bulbs A company produces light bulbs having a life expectancy that is normally distributed with a mean of 2000 hours and a standard deviation of 50 hours.

- **20.** Find the *z*-score for a life expectancy of 2085 hours.
- **21.** What is the probability that a randomly selected light bulb will last at most 2085 hours?



Georgia Performance Standard(s)

> MM3D2b. MM3D3

Your Notes

GRI Approximate Binomial Distributions and Test Hypotheses

Goal • Use normal distributions to approximate binomial distributions.

NORMAL APPROXIMATION OF A BINOMIAL DISTRIBUTION

Consider the binomial distribution consisting of *n* trials with probability p of success on each trial. If $np \ge 1$ and $n(1 - p) \ge 1$, then the binomial distribution can be approximated by a normal distribution with the following mean and standard deviation:

Mean: \bar{x} = Standard deviation: σ =

Example 1 Find a binomial probability

Blood Types About 38% of the U.S. population have blood type 0+. You are conducting a random survey of 400 U.S. residents. What is the probability that at most 162 U.S. residents surveyed have blood type 0+?

The number x of U.S. residents in your survey who have blood type 0 + has a binomial distribution with n =and p =. Approximate the binomial distribution by using a normal distribution.



Checkpoint Complete the following exercise.

 About 34% of the U.S. population have blood type A+. You are conducting a random survey of 250 U.S. residents. What is the probability that at most 78 U.S. residents surveyed have blood type A+? What is the probability that 92 or more U.S. residents surveyed have blood type A+?

HYPOTHESIS TESTING

To test a hypothesis about a statistical measure for a population, use the following steps.

- Step 1 State the hypothesis you are testing. The hypothesis should make a statement about some ______ of a population.
- **Step 2 Collect** data from a random sample of the population and compute the of the sample.
- **Step 3 Assume** the hypothesis is true and calculate the resulting probability *P* of obtaining the sample statistical measure of a more extreme sample statistical measure. If this probability is small (typically P < 0.05), you should _____ the hypothesis.

Example 2 Test a hypothesis

Taxes A recent poll claimed that 25% of U.S. taxpayers file their income tax returns right away. To test this finding, you survey 50 U.S. taxpayers and find that 8 of them file their income tax returns right away. Should you reject the poll's findings? Explain.

- 1. State the hypothesis:
- 2. Collect data and compute a statistical measure. In your survey, 8 out of 50, or ____%, file their income tax returns right away.
- **3. Assume** the hypothesis in Step 1 is true. Calculate the resulting probability of randomly selecting 8 or fewer U.S. taxpayers out of 50 who file their income tax returns right away. The probability is

$$P(x \le 8) = P(x = 0) + P(x = 1) + \cdots + P(x = 8)$$

where each term in the sum is a binomial probability with $n = _$ and $p = _$. You can approximate the binomial distribution by a normal distribution.

$$\overline{x} = np = ___= __$$

$$\sigma = \sqrt{np(1 - p)} =$$
 \approx ____

Using a *z*-score and the standard normal table gives:

$$P(x \leq 8) = P(z \leq$$

So, if it is true that 25% of U.S. taxpayers file their income tax returns right away, then there is about a ____% probability of selecting 8 or fewer U.S. taxpayers who file their income tax returns right away in a random sample of 50 U.S. taxpayers. With this probability, you should _____ the hypothesis.

Homework

Your Notes

Checkpoint Complete the following exercise.

2. Rework Example 2 if 4 of the 50 U.S. taxpayers in the survey file their income tax returns right away.

6.3 Practice

Find the mean and standard deviation of a normal distribution that approximates the binomial distribution with n trials and probability p of success on each trial.

1. n = 20, p = 0.4 **2.** n = 60, p = 0.3 **3.** n = 50, p = 0.2

4.
$$n = 40, p = 0.3$$
 5. $n = 80, p = 0.7$ **6.** $n = 30, p = 0.6$

7.
$$n = 100, p = 0.75$$
 8. $n = 120, p = 0.15$ **9.** $n = 104, p = 0.25$

In Exercises 10–12, use the fact that approximately 9% of U.S. children have asthma. Consider an elementary school with 500 children.

10. What is the probability that at least 45 children have asthma?

- **11.** What is the probability that at most 39 children have asthma?
- **12.** What is the probability that at most 51 children have asthma?

Date ____

6.3 **Practice** continued

In Exercises 13–15, use the fact that 57% of people have never played golf. Consider a random sample of 200 people.

- **13.** What is the probability that 128 or fewer people have never played golf?
- 14. What is the probability that 100 or more people have never played golf?
- **15.** What is the probability that between 107 and 121 people have never played golf?

In Exercises 16–18, state the hypothesis.

16. An opinion poll states that 45% of people choose macaroni and cheese as their favorite food combination.

- **17.** You read an article about phobias. The article claims that 10% of people have a phobia.
- **18.** You are researching colleges. You read in a university's flyer that the school has a 94% job placement rate.

6.4 Select and Draw Conclusions from Samples



Your Notes

Goal • Study different sampling methods for collecting data.

VOCABULARY

Population

Sample

Unbiased sample

Biased sample

Margin of error

Example 1 Classify samples

School Lunch A teacher wants to survey everyone at her school about the quality of the school lunches. Identify the type of sample described and tell if the sample is biased.

- **a.** The teacher surveys every 7th student that goes through the lunch line.
- **b.** From a random name lottery, the teacher chooses 150 students and teachers to survey.
- a. The teacher uses a ______ to select students, so the sample is a ______ sample. This sample is ______ because the teacher surveys the students, but not the teachers.
- b. The teacher chooses from a random lottery, so the sample is a ______ sample. This sample is ______ because both students and teachers are surveyed.

Solution

Your Notes

Checkpoint Identify the type of sample described, and tell whether the sample is biased.

1. A local politician wants to survey his constituents. He mails surveys to the constituents that are members of his political party and uses only the surveys that are returned.

MARGIN OF ERROR FORMULA

When a random sample of size *n* is taken from a large population, the margin of error is approximated by:

Margin of error = \pm

This means that if the percent of the sample responding a certain way is p (expressed as a decimal), then the percent of the population that would respond the same

way is likely to be between p - and p + d

Example 2 Find a margin of error

Newspaper Survey In a survey of 1432 people, 26% said that they read the newspaper every day. (a) What is the margin of error for the survey? (b) Give an interval that is likely to contain the exact percent of all people who read the newspaper every day.

a. Margin of error = $\pm \frac{1}{\sqrt{n}} = \pm \frac{1}{\sqrt{n}} \approx$ _____

The margin of error for the survey is about _____%.

b. To find the interval, add and subtract _____%.

26% - ___% = ___%

26% + % = %

It is likely that the exact percent of all people who read the newspaper every day is between ____% and ____%. Checkpoint Complete the following exercise.

2. In Example 2, suppose the sample size is 3236 people. What is the margin of error for the survey?

Example 3 Find a sample size

Community Survey A group of students survey the local community about their favorite beverages. How many people were surveyed if the margin of error is $\pm 7\%$?

Solution

Use the margin of error formula.

Margin of error
$$= \pm \frac{1}{\sqrt{n}}$$

 $= \pm \frac{1}{\sqrt{n}}$
 $= \pm \frac{1}{\sqrt{n}}$
 $= \frac{1}{n}$
 $n \approx \underline{\qquad}$
About people were surveyed

Checkpoint Complete the following exercise.

3. In a poll about movie channels its customers prefer to watch, a cable company wants the margin of error to be $\pm 3\%$. How many customers would need to be surveyed?

Homework

6.4 Practice

Identify the type of sample described. Then tell if the sample is biased. *Explain* your reasoning.

- **1.** A newspaper is conducting a survey to find out people's favorite sports. The newspaper asks every other person attending a baseball game.
- **2.** A pet store wants to find out people's favorite animals. The store places survey cards at a table for customers to fill out as they enter the store.
- **3.** A health club wants to know how often members attend an aerobics class. The club asks members that have just finished taking an aerobics class.

Find the margin of error for a survey that has the given sample size. Round your answer to the nearest tenth of a percent.

4.	225	5.	100	6.	625	7.	3600
8.	4200	9.	776	10.	390	11.	8000
Find Rou	the sample size ind your answer to	requi the	ired to achieve th nearest whole n	e giv umbe	ven margin of erro er.	or.	
12.	±7%	13.	±6%	14.	±8%	15.	±9%
16	+3.5%	17	+2.2%	18	+4.8%	10	+1 7%



In Exercises 20 and 21, use the following information.

Internet In a survey of 802 people, 16% said that they use the Internet or e-mail more than 10 hours per week.

- **20.** What is the margin of error for the survey? Round your answer to the nearest tenth of a percent.
- **21.** Give an interval that is likely to contain the exact percent of all people who use the Internet or e-mail more than 10 hours per week.

In Exercises 22–24, use the following information.

High School Addition The school board wants to find out how the community feels about a proposed addition to the high school. There are 15,000 people living in the school district. The school board would like to survey 800 people.

- **22.** *Describe* a method for selecting an unbiased, random sample of people who live in the school district.
- **23.** *Describe* a method for selecting people who live in the school district that would be biased.
- **24.** *Describe* a method for selecting people who live in the school district that would not be random.



6.5 Experimental and **Observational Studies**

Goal • Identify types of studies and flaws in experiments.

	Georgia	
	Performance	
1	Standard(s)	
	MM3D3	

Your Notes

Experimental study		
Observational study		

Example 1 *Identify studies*

Tell whether the study is an experimental study or an observational study. Explain your reasoning.

You want to study the effect that listening to music has on the length of time an individual exercises. Each individual in your study runs on a treadmill. The length of time spent running is recorded. The control group is individuals who run on the treadmill without listening to music and the experimental group is individuals who run on the treadmill while listening to music.

Solution

The study is an ______ study because _____

Example 2 Identify flaws in an experiment

Vitamins A researcher conducts an experiment to see if a new vitamin increases a person's energy level. The experimental group consists of college students who are given the vitamin daily. The control group consists of college professors who are not given the vitamin. The researcher finds that the energy level of the people in the experimental group is greater than those in the control group. As a result, the researcher concludes that the vitamin is effective. Identify any flaws in this experiment, and describe how they can be corrected.

On average, college students are likely to be younger than college professors. So, it could be _____ rather than the vitamin that explains why the experimental group had more energy than the control group. To correct this flaw, the researcher could redesign the experiment so that _____

Checkpoint Complete the following exercises.

1. You want to determine if the incumbent Republican candidate for governor is likely to be re-elected. The experimental group consists of registered Republican voters. The control group consists of registered Democrat voters. Tell whether the study is an *experimental study* or an *observational study*. *Explain* your reasoning.

2. You determine from your study in Exercise 1 that a majority of the Republican voters will vote for the incumbent candidate. Identify any flaws in this study and describe how they can be corrected.

Homework

6.5 **Practice**

Determine which group is the experimental group and which group is the control group in the study.

- **1.** A researcher is conducting a study to determine the effect the flu vaccine has on preventing the flu. The participants in the study are a group of people who received the flu vaccine and a group of people who did not receive the flu vaccine.
- **2.** A school district is performing a study to determine whether or not an SAT prep course increases a student's SAT test score. The participants in the study are a group of students who do not take the prep course and a group of students who do take the prep course.
- **3.** A researcher is conducting a study to determine the effects of caffeine on a person's heart rate. The participants in the study are a group of people who drink coffee daily and a group of people who drink coffee occasionally.
- **4.** A lawn care company is conducting a study to determine whether a new lawn treatment is effective in stopping the growth of weeds. The company applies the treatment to only one half of a lawn.

In Exercises 5–8, tell whether the study is an experimental study or an observational study. *Explain* your reasoning.

5. A company wants to study the effect taking a vacation during different times of the year has on a worker's job performance upon returning. Each individual in the study takes a one-week vacation and upon returning, his or her job performance is monitored for one month. The control group consists of workers who choose to take a one-week vacation during the summer. The experimental group consists of workers who choose to take a one-week vacation during the winter.

6.5 **Practice** continued

6. You want to determine whether a name brand laundry detergent works better at cleaning clothes than a generic brand laundry detergent. Each individual in your study is given laundry detergent to clean clothes. You then ask the individuals to complete a survey about how well the detergent cleaned the clothes. The control group consists of individuals who use a generic brand laundry detergent. The experimental group consists of individuals who use a name brand laundry detergent.

7. A scientist wants to study the effect of sunlight on the growth of a plant. The scientist measures the height of each plant in the study after one week. The control group consists of plants that are subjected to 24 hours of darkness. The experimental group consists of plants that are subjected to 12 hours of sunlight and 12 hours of darkness.

8. A golf club manufacturer wants to study the effect a newly designed set of golf clubs has on a golfer's score. The manufacturer lets the participants of the study decide if they want to use new clubs or old clubs. The control group consists of participants who use old clubs. The experimental group consists of participants who use the new clubs.

9. Sunscreen A researcher is conducting a study to determine the effect a new sunscreen has on the prevention of skin cancer. The experimental group consists of lifeguards at an ocean resort. The control group consists of housekeepers at an ocean resort. Identify any flaws in the experiment, and describe how they can be corrected.

Words to Review

Give an example of the vocabulary word.

Random variable	Discrete random variable
Continuous random variable	Probability distribution
Binomial distribution	Binomial experiment
Symmetric	Skewed

Normal distribution	Normal curve
Standard normal distribution	z-score
Population	Sample

Unbiased sample	Biased sample
Margin of error	Experimental study
Observational study	